Signalling, Reputation and Spinoffs

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Internet Appendix

This internet appendix contains proofs which were not included in the main body of the paper. Additionally, it contains other results of interest which were omitted from the paper in order to keep the paper terse. It is organized as follows. In section 1, I discuss a proof which was omitted from the main body of the paper. In particular, I discuss the equilibrium outcome when the worker's type is common knowledge in the baseline model. Section 2 contains results which were not presented in the moral hazard section of the paper - I describe conditions under which the separating equilibrium may not be highest effort equilibrium. In section 3, I discuss conditions under which the separating equilibrium outcome (as described by proposition 1 in the main body of the paper) will be the unique equilibrium outcome of the game.

1 Proof from main body of paper

In the main body of the paper we did not derive the equilibrium outcome when the worker's type is common knowledge in the baseline model. We do so here and show that the worker never forms a spinoff when his type is known. I will solve the one period game beginning in period 2 first and then proceed by backwards induction. Since firm formation is too costly in period two[1], it is trivial to show that the following is true in any equilibrium. If the worker formed his own firm in period one, then he will continue to remain with his own firm in period 2, and if the worker accepted the principal's contract in period one then the principal offers a zero wage contract in period 2 which will be accepted by the worker. Thus, the payoffs in period 2 in any equilibrium can be summarized as follows (first coordinate is the principal's payoffs and the second coordinate is the worker's payoff):

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[1] Both the principal and the worker do not want to invest in firm formation if the reward is just a one period payoff. The assumption of $R_w > R_p > V$ reflects this.

Table 1: Payoffs in Period 2

Table 2: Payoff if contract accepted in Period 1

$(Principal, G)$	$(Principal, B)$
$(V, 0)$	$(\lambda_b V, 0)$

Table 3: Payoff if Spinoff in Period 1

$(Principal, G)$	$(Principal, B)$
$(0, V)$	$(0, \lambda_b V)$

Table 4: Payoff if N in Period 1

$(Principal, G)$	$(Principal, B)$
$(0, 0)$	$(0, 0)$

1.0.1 In Period 1

Claim 1. *In the unique subgame perfect equilibrium of the game, if the worker type is B, the principal offers the worker a contract with zero wages ($\{0, 0\}$) in period 1 and the worker accepts. If the worker type is G then the principal offers the contract $\{2V - R_w, 0\}$ in period 1 and it is accepted by the worker. In period 2, (along the equilibrium path) the principal offers a zero wage contract which is accepted by the worker.*

Proof. By assumption ($2\lambda_b V < R_w$) it is not individually rational (IR) for B worker to invest in forming his own firm in period 1. Therefore, the principal will offer a zero wage contract which will be accepted. Note that it is IR for the principal to offer this contract since $2\lambda_b V > R_p$. It will not be incentive compatible to offer more. When the worker type is G, the worker gets the payoff $(V - R_w + V)$ by investing R_w and starting his own firm. The principal must offer him this amount to dissuade him from leaving. Offering more will not be optimal. Moreover, it is known that the principal cannot commit to offering more than zero wages in period 2 if the worker does not form a firm in period 1. Therefore, the principal offers him a contract which pays him $(2V - R_w)$ upon success [2]. Offering this contract is also IR for the principal as the principal's payoff is $V - (2V - R_w) - R_p + V = R_w - R_p (> 0)$. □

Corollary 1. *The worker never forms his own firm if worker type is known.*

Thus, equilibrium payoffs to the principal and worker in the game when types are known are given by the following matrix where (the first coordinate refers to the principal's payoff)

Table 5: Payoffs if Types are Known

$(Principal, G)$	$(Principal, B)$
$(R_w - R_p, 2V - R_w)$	$(2\lambda_b V - R_p, 0)$

[2]Since G type always succeeds, there is no point in offering him any payoff for failure.

2 Additional Results from Moral Hazard Section

The claims (claim 2 and claim 3) below serve to show that the spinoff equilibrium may not always produce the most effort.

Claim 2. *There exists $\overline{R}$ such that if $R_w > \overline{R}$ and the following holds*

$$\frac{\frac{1+\beta}{1-\beta}+R_w}{1+\beta} < V < \frac{1+R_w}{1+\beta(1-\beta(1-\lambda_b)^2)} < R_p$$

then there exists p' such that if $p_g < p'$, there exists a separating equilibrium where the G worker's strategy (along the equilibrium path) is to leave and form his own firm in period 1 and the B worker's strategy (along the equilibrium path) is to accept a contract in period 1. B type worker exerts $e = 1$ in both periods and G worker exerts $e = 1$ in period 1 and $e = 0$ in period 2. The principal offers the contract $\{(1/(1-\beta)), 0\}$ to the worker in period 1 and the same in period two if the worker accepts in period 1.

Proof. Consider the following strategies in period 2:

For Principal :

$$s_{p,2}(acc, p_g^2) = \{\frac{1}{1-\beta}, 0\} \; ; \; p_g^2 < \frac{V - \frac{1}{(1-\beta)^2} + \frac{\beta(1-\lambda_b)}{(1-\beta)^2}}{\frac{\beta(1-\lambda_b)}{(1-\beta)^2}}$$

$$s_{p,2}(acc, p_g^2) = \{0, 0\} \; ; \; p_g^2 \geq \frac{V - \frac{1}{(1-\beta)^2} + \frac{\beta(1-\lambda_b)}{(1-\beta)^2}}{\frac{\beta(1-\lambda_b)}{(1-\beta)^2}}$$

$$s_{p,2}(N, p_g^2) = \phi$$

$$s_{p,2}(f, p_g^2) = \phi$$

For Worker :

$If\ worker\ had\ formed\ firm\ in\ period\ 1\ only\ one\ case\ is\ relavent\ since\ V < R_p$

$$s_{w,2}(f, p_g^2, \phi, G/B) = S, \; e = 0$$

$If\ worker\ had\ not\ formed\ firm\ in\ period\ 1:$

$$s_{w,2}(acc/N, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, G) = Choose\ (contract, effort)\ maximizing\ payoff$$

$$s_{w,2}(acc/N, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, B) = Choose\ (contract, effort)\ maximizing\ payoff$$

$$s_{w,2}(acc/N, p_g^2, \phi, G/B) = N$$

Consider the following strategies in period 1:

$$\textbf{For Principal}:$$

$$s_p(\phi) = \{\{\frac{1}{1-\beta}, 0\}\}$$

$$\textbf{For Worker}:$$

$$\textit{WLOG let contract 1 be better than contract 2 for G worker}$$

$$s_w(\{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, G) = acc_1, e = 1 \; ; \; -1 + s_1^1 + \frac{1}{1-\beta} = max\{-1 + s_1^1 + \frac{1}{1-\beta}, \beta s_1^1 + \frac{1}{1-\beta}\} \; and$$

$$-1 + s_1^1 + \frac{1}{1-\beta} \geq -1 + V - R_w + \beta V$$

$$s_w(\{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, G) = acc_1, e = 0 \; ; \; -1 + s_1^1 + \frac{1}{1-\beta} \neq max\{-1 + s_1^1 + \frac{1}{1-\beta}, \beta s_1^1 + \frac{1}{1-\beta}\} \; and$$

$$\beta s_1^1 + \frac{1}{1-\beta} \geq -1 + V - R_w + \beta V$$

$$s_w(\{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, G) = L, e = 1 \; ; \; else.$$

$$s_w(\{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, B) = \; Choose \; (contract, effort) \; maximizing \; payoff$$

$$s_w(\phi, B) = N$$

$$s_w(\phi, G) = L, e = 1$$

The belief about the worker's type is formed using Bayesian updating based on equilibrium strategies. On off equilibrium paths, I will assume beliefs consistent with type-independent trembles in the worker's decision[3]. Let us prove by backward induction that the above strategies and beliefs constitute a perfect Bayesian equilibrium under the conditions imposed in the claim.

In period 2, consider the incentives of the worker. If the history is such that the worker has formed a firm of his own in period 1, then the worker is not offered any contracts in period 2 since the principle can never find this to be individually rational ($V < R_p$). In this case, it is clear that the best action for the worker is to play S and then exert zero effort. Zero effort is exerted since the worker gets paid before his effort choice and the game ends in period 2. If the history is one in which the worker had played N in period 1, then it is not individually rational for either the worker or the principal to form a firm in period 1, therefore the only on the equilibrium path response is to play N again. In a history in which the worker had accepted a contract in period 1, it is not individually rational for the worker to play L, therefore the best action choice is to choose the most profitable (contract, effort) tuple.

Now let us consider the incentives of the principal in period 2. Suppose the history is one in which the worker did not accept the principal's contract in period 1. It is not IR for the principal to form the firm in period 2. Therefore, the principal's optimal action is to offer no contract in period 2. If the worker had accepted a contract with the principal in period 1, then the principal has two choices - he can either offer a

[3] If the worker wants to choose an action a, then the worker chooses action a with probability $(1-\varepsilon)$ and any other action with positive probability.

contract $\{(1/(1-\beta)),0\}$ which is the minimum amount required for a worker of either type to put in full effort, or he can offer a contract which pays nothing and get zero effort from the worker. Note that any contract which offers something in the middle is not optimal. We need the following conditions to make it IR and IC for the principal to offer the contract $\{(1/(1-\beta)),0\}$ and extract maximum effort:

$$\begin{aligned}
&Payoff\ from\ \{\frac{1}{1-\beta},0\}\ >\ Payoff\ from\ \{0,0\} \\
&\Leftrightarrow [p_g^2+(1-p_g^2)(\beta\lambda_b+(1-\beta))](V-\frac{1}{1-\beta}) > [p_g^2\beta+(1-p_g^2)\beta\lambda_b]V \\
&\Leftrightarrow p_g^2 \leq \frac{V(1-\beta)^2-\beta\lambda_b-(1-\beta)}{\beta(1-\lambda_b)} \qquad (1)
\end{aligned}$$

It is clear that if R_w is high enough and $V > (((1+\beta)/(1-\beta))+R_w)/(1+\beta)$, then $V(1-\beta)^2-\beta\lambda_b-(1-\beta)>0$. In any separating equilibrium where the G worker forms his own firm in period 1, $p_g^2\approx 0$. Thus, 1 is satisfied.

Therefore, under a separating equilibrium, the payoffs in period 2 are as follows. The principal expects to get a payoff of $(\beta\lambda_b+1-\beta)(V-(1/(1-\beta)))$ in period 2 if the worker accepts the contract in period 1. If the worker does not accept contract in period one, the principal expects zero payoff in period 2. A G type worker expects to get a payoff of βV if he had played L in period 1 and a payoff of $-1+(1/(1-\beta))$ if he had accepted a contract in period 1. A B type worker expects to get a payoff of βV if he had played L in period 1 and a payoff of $-1+(\beta\lambda_b+(1-\beta))\frac{1}{1-\beta}$ if he had accepted a contract in period 1.

Let us now consider the incentives for the worker in period 1. Given the payoffs in period 2, G type worker cannot do better. In any separating equilibrium the G worker must put in maximum effort in period 1 after playing L. We need the following condition to ensure that this is incentive compatible: $V > \frac{1}{\beta(1-\beta)(1-\lambda_b)}$. Given that the G worker will put in full effort after separation, it is IR for the G type worker to choose to play L if $V > \frac{1+R_w}{1+\beta}$. It is clear that if R_w is high enough and $V > \frac{\frac{1+\beta}{1-\beta}+R_w}{1+\beta}$, then these conditions are satisfied. It is not IR for the B type worker to leave if the following condition holds: $V < \frac{1+R_w}{1+\beta(1-\beta(1-\lambda_b)^2)}$. This holds according to the conditions in this claim, therefore, his strategy to choose the best (contract,effort) combination is optimal. Thus, we have that if R_w is high enough and $\frac{1+R_w}{1+\beta} < V < \frac{1+R_w}{1+\beta(1-\beta(1-\lambda_b)^2)}$, then the following hold: 1. IR for G type worker to play L, 2. Not IR for B type worker to play L, 3. If G type worker plays L, then first period effort is 1, 4. B worker will accept $\{\frac{1}{1-\beta},0\}$ and put in effort 1 in both periods, and 5. G worker will play $e=0$ in period 2 after L in period 1.

Consider incentives of G worker in period 1 after the principal offers a contract $\{s,0\}$:

Payoff from accept and $e=1$ = $-1+s+(-1+\frac{1}{1-\beta})$

Payoff from accept and $e=0$ = $\beta s+(-1+\frac{1}{1-\beta})$

Payoff from $L = -1 + V - R_w + \beta V$

Therefore, G worker will accept and play $e = 1$ if

$$s \geq \frac{1}{1-\beta} \ \& \ s \geq V - R_w + (\beta V - \frac{\beta}{1-\beta})$$

G worker will accept and play $e = 0$ if

$$s < \frac{1}{1-\beta} \ \& \ s \geq \frac{1}{\beta}[-1 + V - R_w + (\beta V - \frac{\beta}{1-\beta})]$$

G worker will play L if

$$s < \frac{1}{\beta}[-1 + V - R_w + (\beta V - \frac{\beta}{1-\beta})] \ \& \ s < V - R_w + (\beta V - \frac{\beta}{1-\beta})$$

Now:

$$V > \frac{\frac{1+\beta}{1-\beta} + R_w}{1+\beta}$$

$$\Leftrightarrow V - R_w + (\beta V - \frac{\beta}{1-\beta}) > \frac{1}{1-\beta} \quad (2)$$

and we can show that

$$V - R_w + (\beta V - \frac{\beta}{1-\beta}) < \frac{1}{\beta}[-1 + V - R_w + (\beta V - \frac{\beta}{1-\beta})] \quad (3)$$

So by 2 and 3, if the principal offers $s = \frac{1}{1-\beta}$ or below, G worker will play L. If R_w is high enough then we have $\frac{\frac{1+\beta}{1-\beta}+R_w}{1+\beta} < \frac{1+R_w}{1+\beta(1-\beta(1-\lambda_b)^2)}$. So we can pick a V such that $\frac{\frac{1+\beta}{1-\beta}+R_w}{1+\beta} < V < \frac{1+R_w}{1+\beta(1-\beta(1-\lambda_b)^2)}$.

The principal now has three choices which may be optimal. One, offer a zero wage contract which the B type worker will accept and put zero effort. Two, offer the contract $\{\frac{1}{1-\beta}, 0\}$ which the worker will accept if he is B type and put in full effort. Three, offer the contract $\{V - R_w + (\beta V - \frac{\beta}{1-\beta}), 0\}$ which both type workers will accept and put in full effort[4]. The principal offers the contract $\{\frac{1}{1-\beta}, 0\}$. This is less than the minimum needed to attract the G type worker. Clearly, the principal will be willing to offer no more than this if p_g is low enough i.e. if the principal believes that the market is convinced that the worker must be B type if he accepts a contract, it would not be optimal for the principal to offer a high paying ($V - R_w + (\beta V - \frac{\beta}{1-\beta})$ is strictly bigger than $\frac{1}{1-\beta}$) contract to attract both types of workers[5]. Thus, we can find a p' so that if $p_g < p'$, the principal's optimal choice is to offer $\{\frac{1}{1-\beta}, 0\}$. The principal does not offer a lower pay contract because

[4]Inequation 3 ensures that that offering a contract which would make the G worker accept (and put in zero effort) is not optimal for the principal.

[5]I omit the formal proof of this part since the idea is the same as that in propositions in the main body of the paper.

V is high enough for him to want to induce high effort. Given this contract, the G worker's optimal choice is to form his own firm in period 1. □

Claim 3. *Suppose the conditions required for the previous claim hold. Then, there exists a p'' such that if $p_g \in [0, p'']$, there exists a pooling equilibrium in which a worker of any type chooses to accept a contract in period 1 and 2 and puts in effort=1 in both periods.*

Proof. Suppose the belief about the worker's type on off equilibrium paths are the beliefs consistent with type-independent trembles in the worker's decision. In a pooling on contract equilibrium, a worker of any type is expected to accept the contract. If the worker plays L, it would be viewed by the market as the outcome of a tremble and the worker will have a reputation of p_g. If p_g is low, it would not be individually rational to play L (this follows from $V < \frac{1+R_w}{1+\beta(1-\beta(1-\lambda_b)^2)}$). Therefore, the strategy of both type workers must be to accept the best contract.

Consider the following strategies in period 2:

$$\textbf{For Principal}:$$

$$s_{p,2}(nf, p_g^2) = \{\frac{1}{1-\beta}, 0\} \; ; \; p_g^2 < \frac{V - \frac{1}{(1-\beta)^2} + \frac{\beta(1-\lambda_b)}{(1-\beta)^2}}{\frac{\beta(1-\lambda_b)}{(1-\beta)^2}}$$

$$s_{p,2}(nf, p_g^2) = \{0, 0\} \; ; \; p_g^2 \geq \frac{V - \frac{1}{(1-\beta)^2} + \frac{\beta(1-\lambda_b)}{(1-\beta)^2}}{\frac{\beta(1-\lambda_b)}{(1-\beta)^2}}$$

$$s_{p,2}(N, p_g^2) = \phi$$

$$s_{p,2}(f, p_g^2) = \phi$$

$$\textbf{For Worker}:$$

$$If\ worker\ had\ formed\ firm\ in\ period\ 1\ only\ one\ case\ is\ relavent$$

$$s_{w,2}(f, p_g^2, \phi, G/B) = S, \; e = 0$$

$$If\ worker\ had\ not\ formed\ firm\ in\ period\ 1:$$

$$s_{w,2}(nf/N, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, G) = Choose\ (contract, effort)\ maximizing\ payoff$$

$$s_{w,2}(nf/N, p_g^2, \{\{s_1^2, f_1^2\}, \{s_2^2, f_2^2\}\}, B) = Choose\ (contract, effort)\ maximizing\ payoff$$

$$s_{w,2}(nf/N, p_g^2, \phi, G/B) = N$$

Consider the following strategies in period 1:

$$\textbf{For Principal}:$$
$$s_p(\phi) = \{\{\frac{1}{1-\beta}, 0\}\}$$
$$\textbf{For Worker}:$$
$$s_w(\{\{s_1^1, f_1^1\}, \{s_2^1, f_2^1\}\}, G/B) = \textit{ Choose } (contract, effort) \textit{ maximizing payoff}$$
$$s_w(\phi, G/B) = N$$

The above strategies would give us the pooling on contract equilibrium. The proof is similar to those we have given before so we refrain from reproducing it here. □

Corollary 2. *Given worker type, the above pooling equilibrium is weakly better than the separating equilibrium described in claim 2 since it gives a higher probability of success of the project in each period.*

Proof. For any type of worker, the pooling equilibrium guarantees full effort in both periods. Whereas, the separating equilibrium does not give us full effort from the *G* type worker in period 2. □

3 Uniqueness of Equilibrium

Is it possible to get the equilibrium outcome in the separating spinoff equilibrium[6] as the unique equilibrium outcome? I show that if the following two assumptions hold, and the conditions required in proposition 1 (in the main body of the paper) also holds, then the separating equilibrium outcome is the unique outcome of the game.

- Assumption A - The worker can make mistakes. Formally, in every period, after the worker makes his action decision, nature moves and assigns a positive probability to every action in the worker's choice set. The weight on the actions not chosen by the worker is arbitrarily low.
- Assumption B - Given an equilibrium, if a mistake (non-equilibrium action) gives the worker strictly negative payoffs for all possible beliefs that the market may have about the worker who makes that mistake, then the probability of such a mistake is much lower than the probability of a mistake where the worker may get positive payoffs for some beliefs. In particular, I assume the following mathematical specification[7] - if a mistake gives the worker strictly negative payoffs for all possible beliefs that the

[6] Where the good type worker forms a spinoff in period 1 and then remains at the spinoff in period 2 while the bad type worker's strategy is to accept a contract and stay with the principal in period 2.

[7] Any other specification where one mistake is an order more likely than the other will also work

market may have about the worker who makes that mistake, then the probability of such a mistake is ε^2 (independent of type). Else, the probability of the mistake is ε (independent of type). Thus, in this case, if the worker's strategy is to choose the action b, then nature chooses action b with probability $1-\varepsilon$ and all other actions with equal probability (equal probability weights adding up to ε).

Our equilibrium concept now is extensive form Trembling Hand Perfect Equilibrium. There are several justifications possible for allowing for mistakes. These could be genuine mistakes, experimentation or the result of unobserved shocks. Furthermore, allowing for mistakes tests our equilibrium for robustness.

If Assumption A, assumption B and the conditions required in proposition 1 (in the main body of the paper) hold, then the only equilibrium outcome possible is a separating equilibria where the G worker forms his own firm in period 1 and the B worker's strategy is to accept a contract in period 1, or an equilibrium in which both type worker's accept a contract in period one (as described in claim 2). Suppose there is an equilibrium in which the worker types pool on accepting a contract. If a worker deviates and forms his own firm in period one then the belief about that worker will be close to 1 (ratio of mistake probabilities is $\frac{1}{\varepsilon}$ and $\varepsilon \to 0$). This is because only the G worker can get positive payoffs from forming his own firm. Now, this deviation becomes profitable to the G worker as his payoffs become approximately $2V - R_w$ and therefore we can eliminate equilibria of this kind.

Assumption B is used to restrict beliefs about a worker who makes the costly investment and forms his own firm. Combined with $V < \frac{R_w}{1+\lambda_b(2-\lambda_b)}$, this assumption implies that since only the G type worker can possibly gain by forming his own firm, the beliefs about the worker who plays L in period 1 must be close to 1. This assumption is most natural when the difference between the expected future payoff of a G and B worker is large. If this difference is small, then this assumption becomes a strong one. The difference in expected payoffs is inversely proportional to λ_b. Therefore, assumption B is mild if used when the difference in abilities of the two types are large. The next proposition formally describes the conditions needed to obtain the separating equilibrium as the the unique equilibrium.

Proposition 1. *Let Assumption A and B hold. Let* $V < \frac{R_w}{1+\lambda_b(2-\lambda_b)}$. *If* $p_g < \frac{R_p-\lambda_b R_w+(2\lambda_b V-R_p)}{R_w-\lambda_b R_w+(2\lambda_b V-R_p)}$, *there exists a separating equilibrium in which the G worker leaves to form his own firm in period one and the B worker accepts a zero wage contract in period 1. Moreover, this is the unique equilibrium outcome of the game.*

Proof. $V < \frac{R_w}{1+\lambda_b(2-\lambda)}$ guarantees that the only equilibrium outcomes possible are those in which the G worker leaves and the B worker accepts a contract in period one or those in which both type workers accept a contract in period one. The above discussion eliminates the latter. □